Multiplication and Division

Stage 1

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

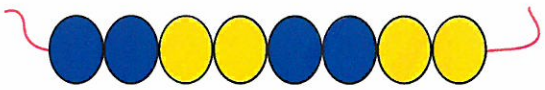
counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

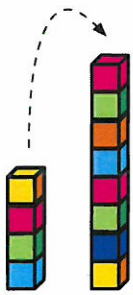
Children will experience practical opportunities involving equal sets or groups using a wide variety of equipment. Practical resources will support children's development of mental pictures and images.

Children will begin to orally count in different multiples including twos, fives and tens making links to natural groupings (e.g. pairs of socks, legs on animals) and the practical resources used.

Children can begin to recognise and continue patterns of multiples using a range of practical resources, e.g. threading beads with two of each colour.

They will begin to use the language and associated representations of doubling.





Double 4 is 8

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÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

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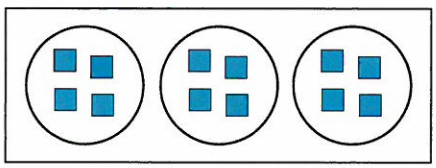
halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___ , ___ equal groups

Children will explore the language of sharing. Children will experience practical activities in 'sharing' objects between a small number of groups/people with the emphasis on sharing equally.

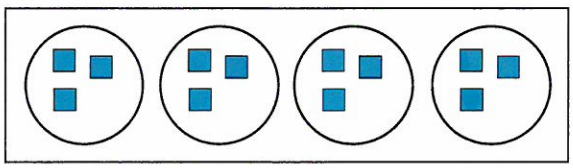
Alongside this, with equal weighting, children should be introduced to 'grouping' objects as a representation of division (e.g. 'each person gets 2') with the emphasis on equal groups.

They will begin to use the language and associated representations of halving.

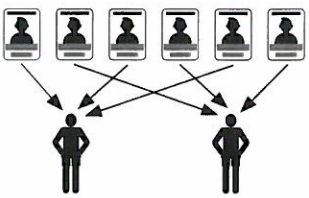
Children can be encouraged to develop ways of recording their findings using pictures.



12 shared into 3 equal groups.
12 shared equally into groups of 4.

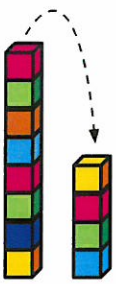


12 shared into 4 equal groups.
12 shared equally into groups of 3.

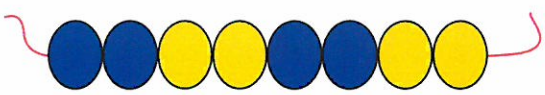


6 football stickers shared between 2 people

6 football stickers, how many people can have 2 each?



Half 8 is 4



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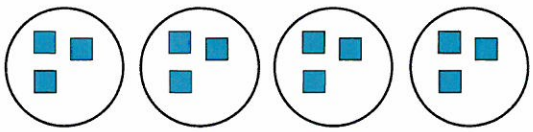
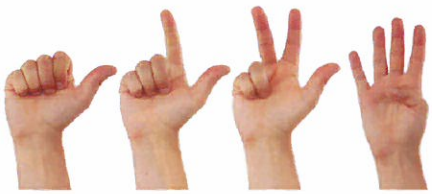
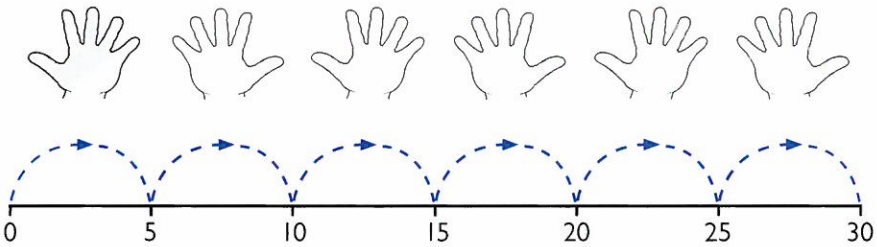
Children will begin to arrange objects into equal groups to aid counting.

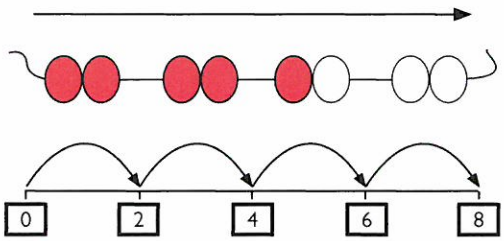


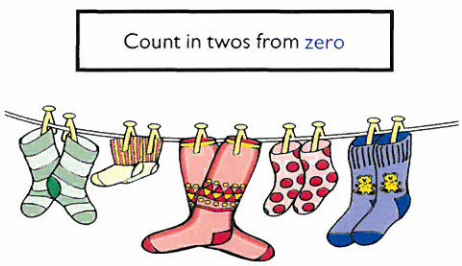
They will continue to count in multiples and begin to relate this to multiplication through finger counting.



Children will be introduced to a variety of representations of repeated addition; they will see the representations alongside each other and begin to make connections between them.







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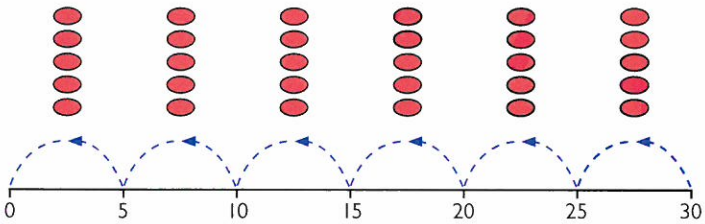
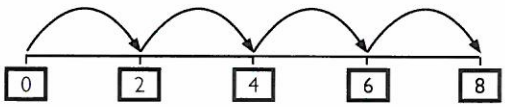
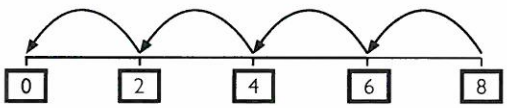
Children will relate the grouping of objects to repeated subtraction and begin to represent this using a number line whilst continuing to use concrete equipment.



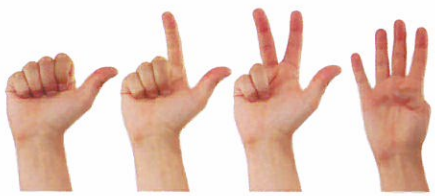


How many 2s are in 8?



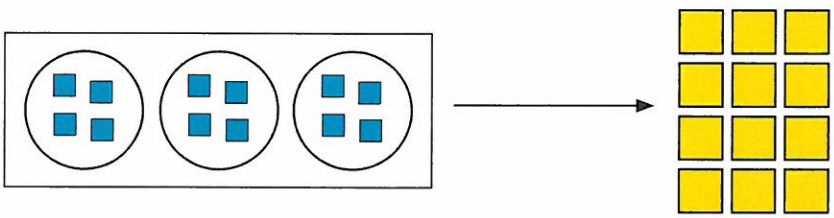
How many 2s can we take away from 8?

Children will use their knowledge of counting up in multiples to solve division calculations and recognise that this in the inverse of multiplication.



"3" "6" "9" "12"

Children will continue to group and share equally using concrete equipment and will now begin to organise their groups into an array rather than scattered groupings.



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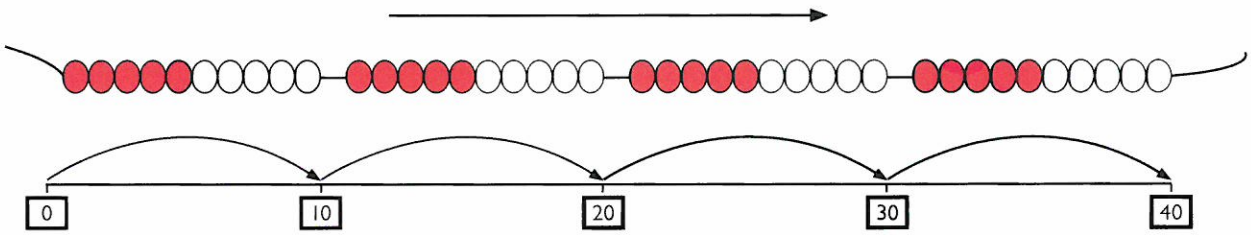
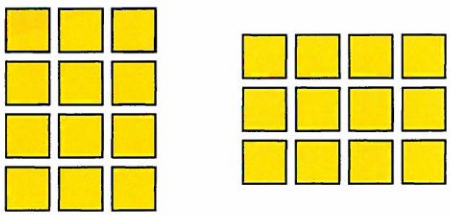
Count in tens from zero



Children will be introduced to the array, using concrete equipment, for small numbers as a way of organising groups to show repeated addition and **commutativity**. They should explore arrays in the world around us, e.g. egg boxes, baking trays, wrapping papers; and use them to answer questions such as 'How many eggs would we need to fill the egg box?' 'How do you know?'



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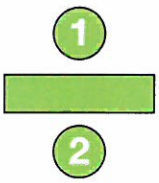
The direct link between multiplication and division should be made explicit when using models and representations.

Children will continue to make links between division and fractions. They will be aware that the division sign is the equivalent to the fraction line and so p÷q can be written as $\frac{p}{q}$.

1 ÷ 2



Stage 3

POINTS TO REMEMBER

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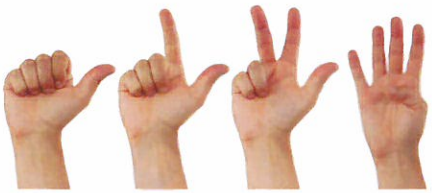
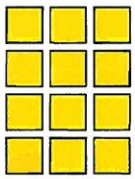
Children will continue to count in multiples and relate this to multiplication through finger counting.



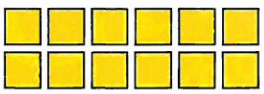
"7" "14" "21" "28"

They will be able to model a calculation using a practical array which demonstrates an effective method of counting and the link to repeated addition. Children need to explore related multiplication facts of a given number by making a variety of arrays and explaining what they show.

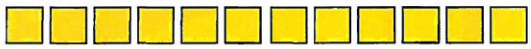
Representing 12



3 x 4 = 12
4 x 3 = 12



2 x 6 = 12
6 x 2 = 12

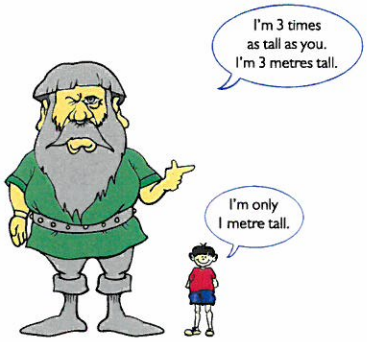


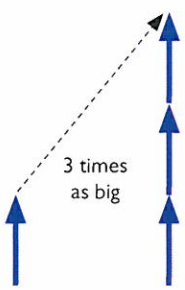
1 x 12 = 12
12 x 1 = 12

3 + 3 + 3 + 3 + 3 = 15

5 + 5 + 5 = 15

The children should be confident with their use of the language of scaling when talking about multiplication.





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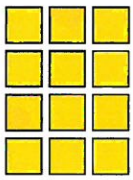
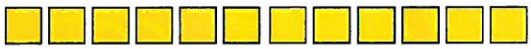
Children will continue to use their knowledge of counting in multiples to support the inverse of multiplication and repeated subtraction.

Children will build on their use of concrete arrays for division recognising the links to repeated subtraction and the inverse of multiplication in order to derive the associated division facts. Children need to explore related division facts of a given number by making a variety of arrays and explaining what they show.

Representing 12

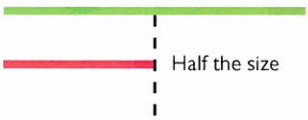




12 into ____ equal groups gives ____ in each group

12 into equal groups of ____ gives ____ groups

The children should be confident with their use of the language of scaling when talking about division with links made to simple fractions (e.g. half the size, three times smaller).



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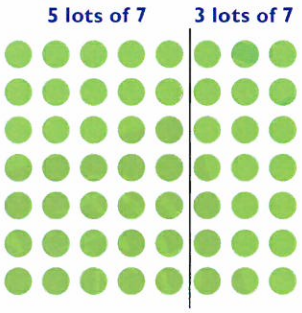
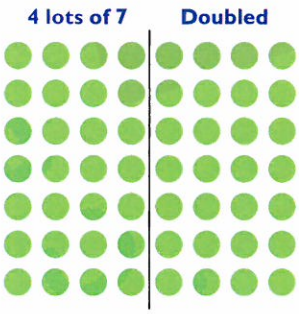
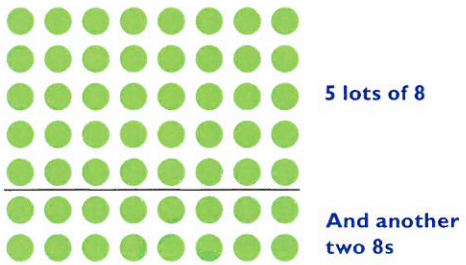
Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays and will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them efficiently calculate totals within arrays. E.g. for 7x8... Children may find counting in 7s or 8s tricky but they can look for 'friendly' numbers which are easier to calculate e.g. 4x5, 4x2, 4x5, 4x2.

Thinking flexibly about 7 x 8







Children should continue to experience the language of scaling (e.g. scaling up pictures by multiplying by powers of 10, multiplying by powers of 1000 in converting between units of measure)

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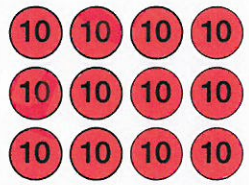
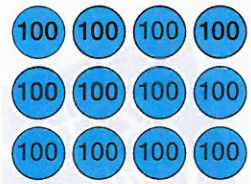
Children will continue to organise groups into an array now working with larger numbers by either grouping or sharing. Children will be able to explain all the facts they know about a given array with no remainder. They should be making arrays with the equipment to establish 'How many in each group?' or 'How many groups?'. Children should continue to experience the language of scaling (e.g. scaling down pictures by dividing by powers of 10, dividing by powers of 1000 in converting between units of measure)

120 ÷ 3



120 shared equally between 3 is 40.
120 shared equally between 4 is 30.
3 equal groups of 40 make 120.
4 equal groups of 30 make 120.

1200 ÷ 3



1200 shared equally between 3 is 400.
1200 shared equally between 4 is 300.
3 equal groups of 400 make 120.
4 equal groups of 300 make 1200.

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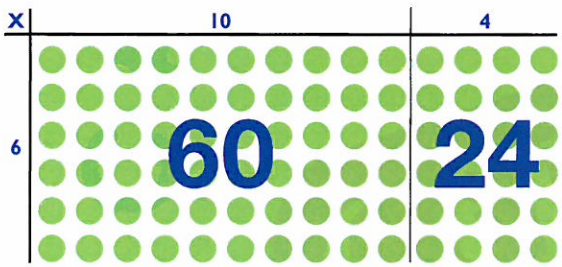
Children will continue to work with arrays, exploring larger numbers, leading into the grid method of multiplication. Practical experiences may still be required for some children as they enter this stage. To begin with, children should see the array with the grid lines. When appropriate, children should move to using the grid displaying the numbers only.

Children should begin using grid method for 2- and 3- digit by 1 digit numbers and should be given the chance to relate this to facts they know about arrays where needed.

Throughout this stage, children should be encouraged to estimate an approximate answer in order to check for reasonableness and this should become standard practice.



$(6 \times 10) + (6 \times 4)$

$60 + 24$

84

x	10	4
6	60	24

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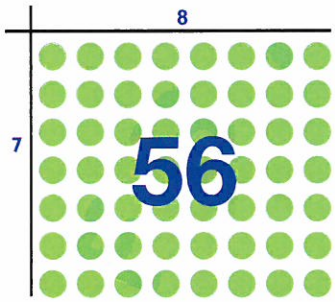
Children will continue to work with concrete arrays, exploring known multiplication/division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the array within short division can be interpreted for both sharing between or equal groups of where the dots within the array each represent 1.



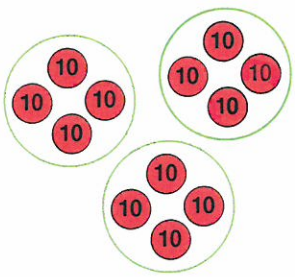
How many equal groups of 7 can I make?
(grouping is represented in the columns)

or

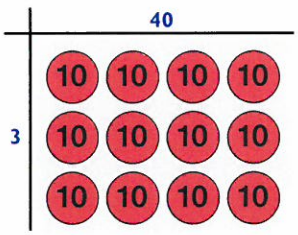
If I put these into 7 equal groups, how many in each group?
(sharing between is represented in the rows)

Children will begin to use counters within an array to show the sharing model of division, using their knowledge of the principle of exchange where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.

120 can be exchanged for 12 tens in order to make an array



120 shared into 3 equal groups gives 40 in each group



We can explicitly see 40 three times; 3 rows of 40, a ⅓ of 120 is 40.
We can divide the array into three parts and there is 40 in each part.

POINTS TO REMEMBER

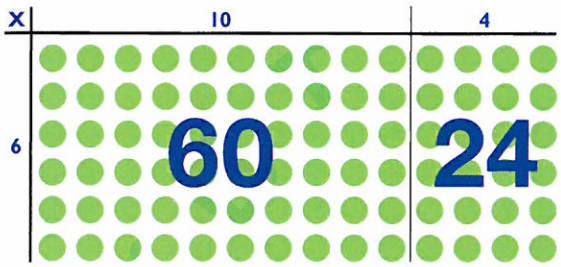
- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

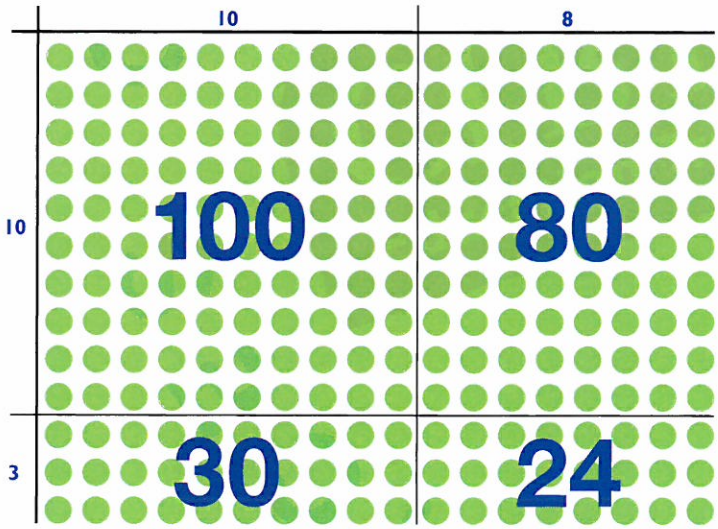
counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will now be secure in using the grid method for multiplying by one-digit numbers and will begin to explore the links between the grid method and the expanded method of short multiplication.



$$\begin{array}{rr} & 1\ 4 \\ \times & 6 \\ \hline & 2\ 4 \quad (6 \times 4) \\ + & 6\ 0 \quad (6 \times 10) \\ \hline & 8\ 4 \end{array}$$

Children will also begin to explore the use of arrays and the grid method for multiplying by two-digit numbers.



x	10	8
10	100	80
3	30	24

$$\begin{array}{rrr} & 1 & 8\ 0 \\ + & & 5\ 4 \\ \hline & 2 & 3\ 4 \\ \hline & 1 & \end{array}$$

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

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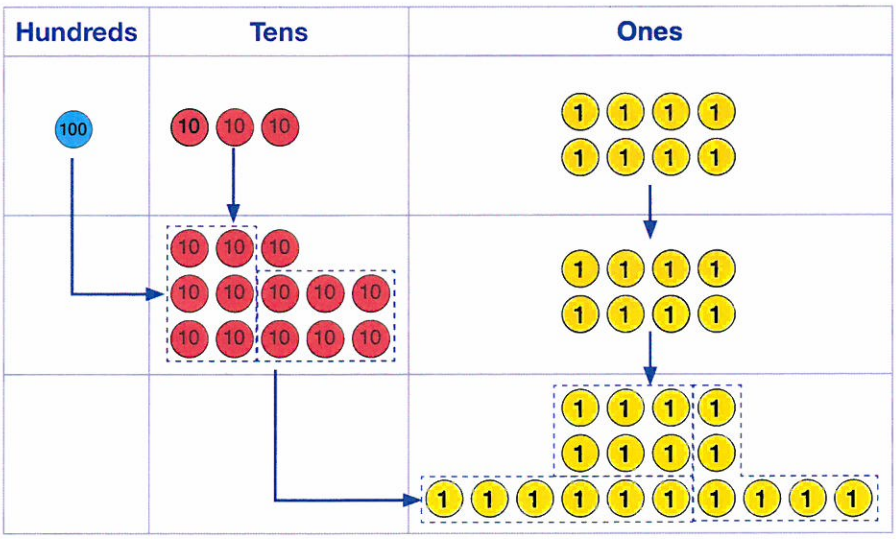
halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___ , ___ equal groups

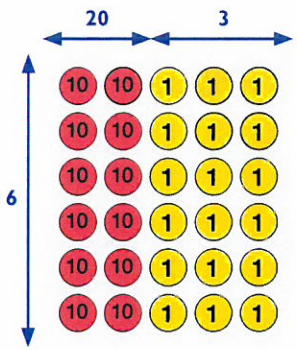
Children will work with equipment to divide any integer by a single digit divisor using their sound knowledge of the principle of exchange.

They will begin to be introduced to numbers that have remainders and will recognise and be able to talk about these when they do not 'fit' into the array.

Children will be introduced to the notation of short division, linking with the principle of exchange and how this relates to the practical representations.

Children continue to be encouraged to consider the links between the sharing model and fractions.





In the array, we can explicitly see 23 six times; 6 rows of 23. This is the sharing model.

⅙ of 138 is 23.

We can divide the array up into six equal parts and there is 23 in each part.

$$\begin{array}{r|l} & 2\ 3 \\ \hline 6 & \not{1}\ {}^{1}3\ {}^{1}8 \end{array}$$

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will now have a good understanding of the expanded short multiplication method and will begin to represent this as compact short multiplication for TU x U.

```
  1 4
x   6
-----
  2 4   (6 x 4)
+ 6 0   (6 x 10)
-----
  8 4
```

→

```
  1 4
x   6
-----
  8 4
  2
```

Children will be secure in using the grid method for multiplying by two-digit numbers and will begin to explore the links between the grid method and the expanded method of long multiplication.

x	10	8
10	100	80
3	30	24

→

```
    1 8
  x 1 3
-------
    2 4   (3 x 8)
    3 0   (3 x 10)
    8 0   (10 x 8)
+ 1 0 0   (10 x 10)
-------
  2 3 4
-------
  1
```

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
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÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

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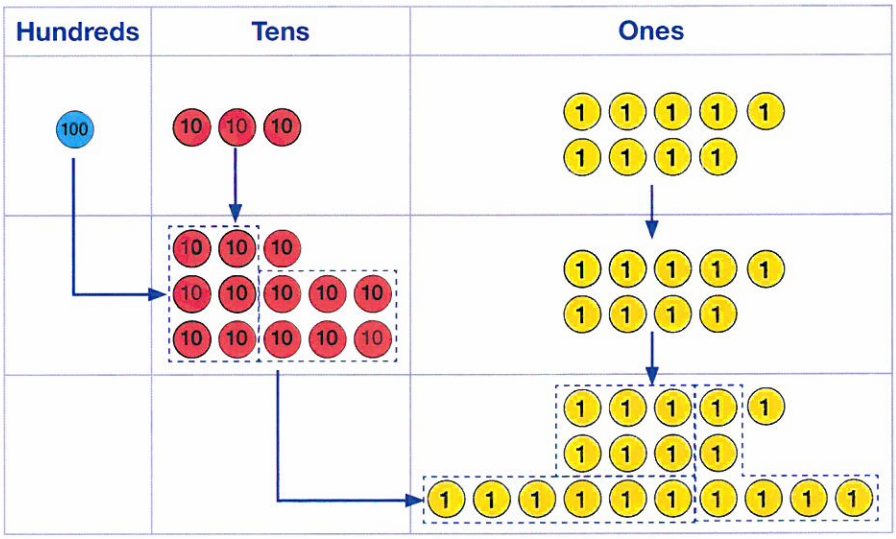
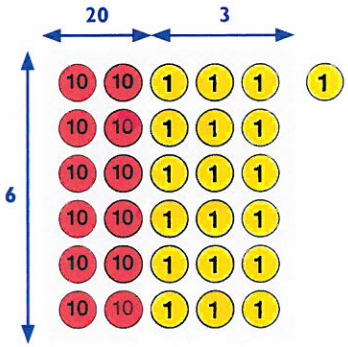
Children will now be secure in using short division for one-digit divisors with an integer quotient.

They will now begin to use the short division notation for calculations involving remainders.





$$\begin{array}{r} 2\ 3\ \text{r}1 \\ 6\overline{)\ \not{1}\ {}^{1}3\ {}^{1}9} \end{array}$$

Children will also begin to explore the use of jottings of friendly numbers to support long division of calculations with 2-digit divisors.

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

20 x 15 = 300

$$\begin{array}{rr} & 2\ 8 \\ 15\overline{)} & 4\ 2\ 0 \\ - & 3\ 0\ 0 \quad (20 \times 15) \\ \hline & 1\ 2\ 0 \\ - & 1\ 2\ 0 \quad (8 \times 15) \\ \hline & 0 \end{array}$$

Stage 8

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
- Use the language '**digit**' not number (number is the amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

VOCABULARY **Ensure the correct vocabulary is used at all stages of learning**

counting, steps, each, doubling, scaling, times, twice as big, ____ times as big, count in ones, count in ____, lots of, groups of, ×, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide... and so on), repeated addition, array, row, column, double, group in pairs, threes... tens, equal groups of, multiplication, product, inverse

Children will now have a good understanding of the short multiplication method.

Children will now have a good understanding of the expanded long multiplication method and will begin to represent this as compact long multiplication.

```
    1 8
  x 1 3
  -----
    2 4   (3 x 8)
    3 0   (3 x 10)
    8 0   (10 x 8)
+ 1 0 0   (10 x 10)
  -----
  2 3 4
  -----
    1
```

→

```
      2
    1 8
  x 1 3
  -----
    5 4
    3 0
+ 1 8 0
  -----
  2 3 4
  -----
    1
```

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

Stage 8

POINTS TO REMEMBER

- Use the language '**calculation**' not 'sum' ('sum' means 'plus or 'total'.)
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÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷

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halve, share, share equally, one each, two each, three each..., divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___ , ___ equal groups

Children will now be secure in using short division for one-digit divisors and long division for two-digit divisors with an integer quotient.

They will now explore the use of long division for two-digit divisors which may include a remainder.

The children will begin to interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
8 x 15 = 120
10 x 15 = 150

20 x 15 = 300

```
      2  8 r12
15 | 4  3  2
   - 3  0  0   (20 x 15)
     1  3  2
   - 1  2  0   (8 x 15)
        1  2
```

```
      2  8 12/15
15 | 4  3  2
   - 3  0  0   (20 x 15)
     1  3  2
   - 1  2  0   (8 x 15)
        1  2
```

```
      2  8 4/5
15 | 4  3  2
   - 3  0  0   (20 x 15)
     1  3  2
   - 1  2  0   (8 x 15)
        1  2
```

Calculating with decimals

When working with decimals, the above stages should always be followed to allow for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.